**b) Run the MC program again with data from Batches 1 and 2. Experiment with different value of NT (time steps) and NSIM (simulations or draws). In particular, how many time steps and draws do you need in order to get the same accuracy as the exact solution? How is the accuracy affected by different values for NT/NSIM?**

The test results for batches 1 and 2 will be shown in the following table. Since the Call and Put options are the same for Batch 2, we can do some less experiments.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Batch | Type | Exact | NT | NSIM | Answer | Error |
| 1 | Call | 2.13337 | 10 | 100000 | 2.11117 | 2.22E-2 |
| 1 | Call | 2.13337 | 50 | 100000 | 2.10652 | 2.68E-2 |
| 1 | Call | 2.13337 | 100 | 100000 | 2.13043 | 2.94E-3 |
| 1 | Call | 2.13337 | 200 | 100000 | 2.14888 | 1.55E-2 |
| 1 | Call | 2.13337 | 10 | 500000 | 2.10919 | 2.42E-2 |
| 1 | Call | 2.13337 | 50 | 500000 | 2.13383 | 4.60E-4 |
| 1 | Call | 2.13337 | 100 | 500000 | 2.14037 | 7.00E-3 |
| 1 | Call | 2.13337 | 200 | 500000 | 2.13440 | 1.03E-3 |
| 1 | Call | 2.13337 | 500 | 500000 | 2.12530 | 8.07E-3 |
| 1 | Call | 2.13337 | 10 | 1000000 | 2.11674 | 1.66E-2 |
| 1 | Call | 2.13337 | 50 | 1000000 | 2.13323 | 1.40E-4 |
| 1 | Call | 2.13337 | 100 | 1000000 | 2.13271 | 6.60E-4 |
| 1 | Call | 2.13337 | 200 | 1000000 | 2.13356 | 1.90E-4 |
| 1 | Call | 2.13337 | 500 | 1000000 | 2.13071 | 2.86E-3 |
| 1 | Call | 2.13337 | 10 | 2000000 | 2.11887 | 1.45E-2 |
| 1 | Call | 2.13337 | 50 | 2000000 | 2.13074 | 2.63E-3 |
| 1 | Call | 2.13337 | 100 | 2000000 | 2.13115 | 2.22E-3 |
| 1 | Call | 2.13337 | 200 | 2000000 | 2.13343 | 6.00E-5 |
| 1 | Call | 2.13337 | 500 | 2000000 | 2.13428 | 9.10E-4 |
| 1 | Put | 5.84628 | 10 | 100000 | 5.84560 | 6.80E-4 |
| 1 | Put | 5.84628 | 50 | 100000 | 5.85634 | 1.01E-2 |
| 1 | Put | 5.84628 | 100 | 100000 | 5.87321 | 2.69E-2 |
| 1 | Put | 5.84628 | 200 | 100000 | 5.85243 | 6.15E-3 |
| 1 | Put | 5.84628 | 10 | 500000 | 5.83997 | 6.31E-3 |
| 1 | Put | 5.84628 | 50 | 500000 | 5.83961 | 6.67E-3 |
| 1 | Put | 5.84628 | 100 | 500000 | 5.84408 | 2.20E-3 |
| 1 | Put | 5.84628 | 200 | 500000 | 5.85466 | 8.38E-3 |
| 1 | Put | 5.84628 | 500 | 500000 | 5.85493 | 8.65E-3 |
| 1 | Put | 5.84628 | 10 | 1000000 | 5.84074 | 5.54E-3 |
| 1 | Put | 5.84628 | 50 | 1000000 | 5.84020 | 6.08E-3 |
| 1 | Put | 5.84628 | 100 | 1000000 | 5.85125 | 4.97E-3 |
| 1 | Put | 5.84628 | 200 | 1000000 | 5.85816 | 1.19E-2 |
| 1 | Put | 5.84628 | 500 | 1000000 | 5.84125 | 5.03E-3 |
| 1 | Put | 5.84628 | 10 | 2000000 | 5.83239 | 1.39E-2 |
| 1 | Put | 5.84628 | 50 | 2000000 | 5.84793 | 1.65E-3 |
| 1 | Put | 5.84628 | 100 | 2000000 | 5.85318 | 5.10E-3 |
| 1 | Put | 5.84628 | 200 | 2000000 | 5.84835 | 2.07E-3 |
| 1 | Put | 5.84628 | 500 | 2000000 | 5.83901 | 7.27E-3 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Batch | Type | Exact | NT | NSIM | Answer | Error |
| 2 | Call | 7.96557 | 10 | 100000 | 7.96130 | 4.27E-3 |
| 2 | Call | 7.96557 | 50 | 100000 | 7.90290 | 6.27E-2 |
| 2 | Call | 7.96557 | 100 | 100000 | 7.94362 | 2.20E-2 |
| 2 | Call | 7.96557 | 200 | 100000 | 8.00060 | 3.50E-2 |
| 2 | Call | 7.96557 | 10 | 500000 | 7.94988 | 1.57E-2 |
| 2 | Call | 7.96557 | 50 | 500000 | 7.97618 | 6.11E-3 |
| 2 | Call | 7.96557 | 100 | 500000 | 7.98116 | 1.56E-2 |
| 2 | Call | 7.96557 | 200 | 500000 | 7.96235 | 3.22E-3 |
| 2 | Call | 7.96557 | 500 | 500000 | 7.94180 | 2.38E-2 |
| 2 | Call | 7.96557 | 10 | 1000000 | 7.96922 | 3.65E-3 |
| 2 | Call | 7.96557 | 50 | 1000000 | 7.97678 | 1.12E-2 |
| 2 | Call | 7.96557 | 100 | 1000000 | 7.96250 | 3.07E-3 |
| 2 | Call | 7.96557 | 200 | 1000000 | 7.96113 | 4.44E-3 |
| 2 | Call | 7.96557 | 500 | 1000000 | 7.96142 | 4.15E-3 |
| 2 | Call | 7.96557 | 10 | 2000000 | 7.97851 | 1.29E-2 |
| 2 | Call | 7.96557 | 50 | 2000000 | 7.96661 | 1.04E-3 |
| 2 | Call | 7.96557 | 100 | 2000000 | 7.95934 | 6.23E-3 |
| 2 | Call | 7.96557 | 200 | 2000000 | 7.96492 | 6.50E-4 |
| 2 | Call | 7.96557 | 500 | 2000000 | 7.97186 | 6.29E-3 |

After we have done the experiments, we can find that in particular, if the number of time step is more than 100 and the number of draws is more than 1,000,000, then we can guarantee the accuracy to two places behind the decimal point (less than 1E-2).

When NSIM increases, that is the draws increases, the accuracy will increase because of the law of big numbers. When NT increases, the accuracy will not necessarily increase. For example some of the samples show their best performance when NT=100 or 200, rather than 500. However, the time step cannot be too small.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Batch | Type | Exact | NT | NSIM | Answer | Error |
| 4 | Call | 92.17570 | 100 | 100000 | 89.4248 | 2.75 |
| 4 | Call | 92.17570 | 200 | 100000 | 92.2125 | 3.68E-2 |
| 4 | Call | 92.17570 | 500 | 100000 | 93.4061 | 1.23 |
| 4 | Call | 92.17570 | 1000 | 100000 | 92.7001 | 5.24E-1 |
| 4 | Call | 92.17570 | 100 | 500000 | 89.5692 | 2.60 |
| 4 | Call | 92.17570 | 200 | 500000 | 91.2852 | 8.91E-1 |
| 4 | Call | 92.17570 | 500 | 500000 | 91.8580 | 3.18E-1 |
| 4 | Call | 92.17570 | 1000 | 500000 | 91.9841 | 1.92E-1 |
| 4 | Call | 92.17570 | 100 | 1000000 | 89.5241 | 2.65 |
| 4 | Call | 92.17570 | 200 | 1000000 | 91.3077 | 8.68E-1 |
| 4 | Call | 92.17570 | 500 | 1000000 | 91.8450 | 3.31E-1 |
| 4 | Call | 92.17570 | 1000 | 1000000 | 91.5646 | 6.11E-1 |
| 4 | Call | 92.17570 | 100 | 2000000 | 89.2492 | 2.93 |
| 4 | Call | 92.17570 | 200 | 2000000 | 90.9017 | 1.27 |
| 4 | Call | 92.17570 | 500 | 2000000 | 91.7973 | 3.78E-1 |
| 4 | Call | 92.17570 | 1000 | 2000000 | 91.7430 | 4.33E-1 |
| 4 | Put | 1.24750 | 100 | 100000 | 1.29604 | 4.85E-2 |
| 4 | Put | 1.24750 | 200 | 100000 | 1.27242 | 2.67E-2 |
| 4 | Put | 1.24750 | 500 | 100000 | 1.25376 | 6.26E-3 |
| 4 | Put | 1.24750 | 1000 | 100000 | 1.25990 | 1.24E-2 |
| 4 | Put | 1.24750 | 100 | 500000 | 1.28476 | 3.73E-2 |
| 4 | Put | 1.24750 | 200 | 500000 | 1.26809 | 2.06E-2 |
| 4 | Put | 1.24750 | 500 | 500000 | 1.25868 | 1.12E-2 |
| 4 | Put | 1.24750 | 1000 | 500000 | 1.25077 | 3.27E-3 |
| 4 | Put | 1.24750 | 100 | 1000000 | 1.29275 | 4.53E-2 |
| 4 | Put | 1.24750 | 200 | 1000000 | 1.27043 | 2.29E-2 |
| 4 | Put | 1.24750 | 500 | 1000000 | 1.25428 | 6.78E-3 |
| 4 | Put | 1.24750 | 1000 | 1000000 | 1.24861 | 1.11E-3 |
| 4 | Put | 1.24750 | 100 | 2000000 | 1.29321 | 4.57E-2 |
| 4 | Put | 1.24750 | 200 | 2000000 | 1.26716 | 1.97E-2 |
| 4 | Put | 1.24750 | 500 | 2000000 | 1.25361 | 6.11E-3 |
| 4 | Put | 1.24750 | 1000 | 2000000 | 1.24929 | 1.79E-3 |

1. **c) Now we do some stress-testing of the MC method. Take Batch 4. What values do we need to assign to NT and NSIM in order to get an accuracy to two places behind the decimal point? How is the accuracy affected by different values for NT/NSIM?**

The test results are shown above. We can see that it is harder for Batch 4 to reach the accuracy. For Call options it cannot reach the accuracy, and for Put option it requires NT=1000 and NSIM=1,000,000. We can see in the tests that now the time step NT is important. If NT is too small, it cannot reach accuracy. This is probably because the expiry date is long. Also, when NSIM increases, the accuracy typically increases.